GUIDED WAVE ENERGY DISTRIBUTION ANALYSIS IN INHOMOGENEOUS PLATES

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INTRODUCTION

An analysis of guided wave energy propagation in a inhomogeneous multi-layered composite structure is presented. It has earlier been reported that ultrasonic guided wave energy within an inhomogeneous composite materials tend to follow the orientation of the fibers, even when the plane of the incident wave is not along the fiber direction [1-3]. In this paper, an exact analysis of a plane wave incident onto a generally anisotropic, visco-elastic, multi-layered composite structure is use here to study the energy flow behavior of guided ultrasonic waves in inhomogeneous composite plates [4] is used to predict this behavior. The reflected and refracted coefficients are obtained by using the well know Thomson-Haskell transfer matrix method [5] which transfer boundary conditions from one side of a solid medium to the other. Then, the power flow vector is used to study the energy distributions within the plates as well as the generation mechanism of guided waves.

This work also show that the assumption of the superposition of partial waves, used in the derivation of dispersion relationships for anisotropic plates, [6] may not be valid for the computation of the power flow vector (the magnitude represents energy), especially in cases when the plane of incidence is not along the planes of material symmetry. This is caused due to the splitting of the fundamental wave mode propagation energy along preferred directions, [7] thus not satisfying the superposition principles. Theoretical results will be supported through experimental results on guided plate wave flow patterns.

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THEORETICAL FORMULATION

Let us first consider the plane wave propagation in a generally anisotropic multi-layered structure as shown in Fig.1. The wave is assumed with the displacements in the form

$$u_i = AU_i e^{j(k_i x_i - \omega t)} \qquad (1)$$

where i=1,2,3; A is the amplitude; U_i is the displacement vector; k_i are the wave number's cosine components; ω is the circular frequency. Substituting eq.(1) into the equation of motion

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{1}{2} C_{ijkl} \frac{\partial}{\partial x_j} \left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right) \qquad (2)$$

gives a system of equations

$$(\Gamma_{ij} - \delta_{ij} \rho \omega^2) U_i = 0 \qquad (3)$$

where, $\Gamma_{ij} = k_l k_m C_{ijlm}$, and C_{ijlm} are material constants which in a general form are complex, ie.

$$C_{ijlm} = C^1_{ijlm} + j\omega\eta_{ijlm} \qquad (4)$$

Here, η_{ijlm} are viscosity coefficients. Since the incident direction is known, the coordinate system can be chosen by considering x_3 normal to the interface between the layers and x_1 within the incident plane. Thus k_2=0 and k_1 can be evaluated. Since the unit displacement vector $U_i \neq 0$, the determinant of the coefficient matrix equals zero as represented below:

$$\det(\Gamma_{ij} - \delta_{ij} \rho \omega^2) = 0 \qquad (5)$$

Expanding the above equation a 6th order of polynomial expression is obtained

$$a_0 k_3^6 + a_1 k_3^5 + a_2 k_3^4 + a_3 k_3^3 + a_4 k_3^2 + a_5 k_3 + a_6 = 0, \qquad (6)$$

Where a_i (i=0,1,2,3,4,5,6) are functions of the material properties. Solving eq. (6), the six roots of k_3 can be found. In general, these roots are complex

$$k_3 = k_3' + j k_3'' \qquad (7)$$

where k'_3 defines the wave propagation direction and k''_3 defines the attenuation of the wave amplitude in the x_3 direction. Substituting k_3 to Eq.(3) one can get six unit displacement vectors U_i^α with respect to six k_3^α(α=1,2,...,6), then substitute k_3 and U_i^α to eq.(1) and using supposition, the displacement field is obtained.Then by using the Thomson transfer matrix, the displacements can be obtained

$$s_i = T_{ij} s_j' \qquad (8)$$

where $s'_i = [u'_1, u'_2, u'_3, \sigma'_{33}, \sigma'_{23}, \sigma'_{13}]^T$ is the displacements and stresses at the top interface, and $s_i = [u_1, u_2, u_3, \sigma_{33}, \sigma_{23}, \sigma_{13}]^T$ is the displacements and stresses inside plate at depth 'z'. T_{ij} is the

Thomson transfer matrix.

It is known that the S_i is generally a combination of two quasi-longitudinal partial wave modes and 4 quasi-shear partial wave modes. Hence, we get the following relationship which must be solved to obtain the reflection and transmission factors for the amplitude of the six partial wave modes A^α (where α=1,2,...6) which represent the reflection and transmission factor amplitudes. The reflection coefficient and transmission coefficient of power flow (energy) are calculated after obtaining the amplitude reflection and transmission coefficients (A^α). Thus, the displacements corresponding to each wave can be obtained as below :

$$u_i^\alpha = A^\alpha U_i^\alpha \exp j(K_1 x_1^\alpha - \omega t) \tag{9}$$

After getting the particle displacement field, the particle velocity can easily be obtained through applying derivative with respect to 't' and the stress field can be obtained from Hook's law as shown below

$$v_i = -j\omega u_i \tag{10}$$

and

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \tag{11}$$

The power flow (energy reflection and transmission coefficients) using the superposition of partial waves is then defined as [1]

$$P_i = \frac{1}{2}\sigma_{ij} v_j^* \tag{12}$$

and the power flow with respect to each partial wave mode is provided by

$$P_i^\alpha = \frac{1}{2}\sigma_{ij}^\alpha v_j^\alpha * \tag{13}$$

RESULTS AND DISCUSSION

The theoretical model was used to analyze the magnitude (energy) and propagation direction of power flow vector. The 3 layer graphite epoxy composite case studies will be examined at a frequency around 1 MHz. The experiments were conducted using the setup illustrated in Fig. 2 and the plate wave flow patterns (PWFP) thus obtained are well discussed in earlier publications [1-3]. Both, the power flow vectors using superposition as well as without superpositions, were analyzed. The results obtained using superposition were unable to explain the PWFP experimental results and hence will not be presented in this paper. Instead, the results for the power flow (energy) of the individual partial wave modes for three graphite-epoxy composite specimen (C, D, and E) are presented in Figs. 3-5.

In Figure 3a, the magnitude of the power flow vector (energy) for specimen C with $\{45_5/135_5\}_S$ ply layup is presented as a function of distance into the specimen (x_3). The three ply groups (upper - {45 degree}, lower - {135 degree}, and the bottom - {45 degree}) are

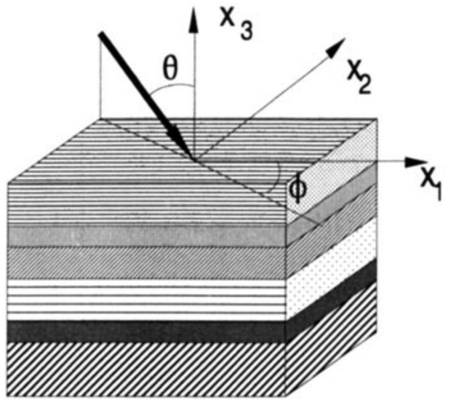


Figure 1. Representation of the theoretical multilayered model.

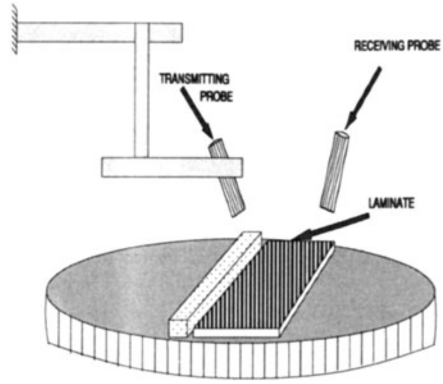
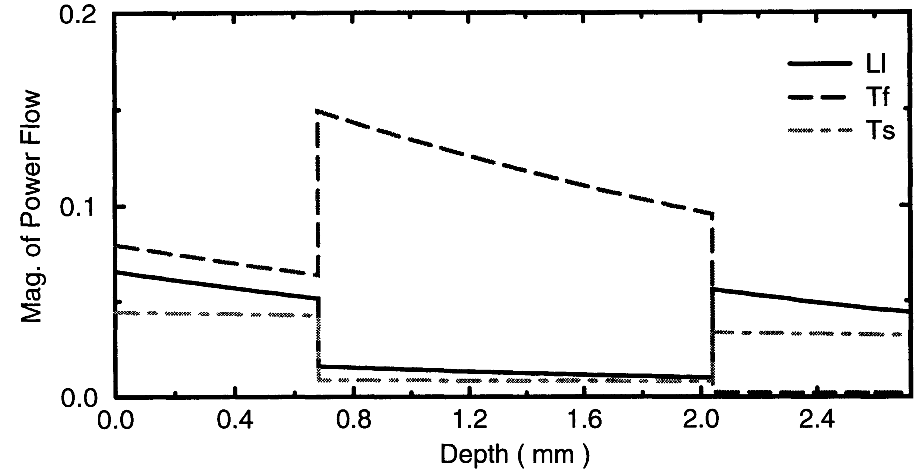


Figure 2 . Plate Wave Flow Pattern method. [1]

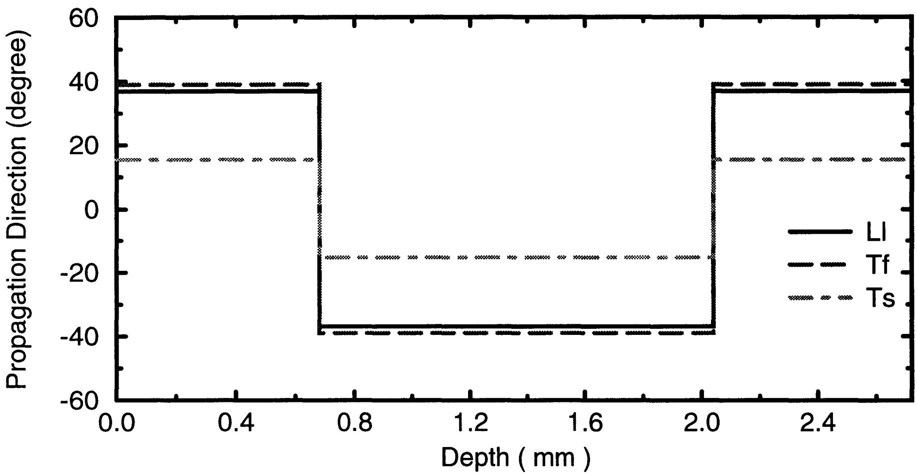
clearly distinguishable. Here Ll represents the two partial modes of quasi-longitudinal bulk wave modes (dark lines are partial modes with wave vector pointing down and light lines represent the upward pointing partial modes). Similiarly, Tf is the fast quasi-transverse partial wave modes while Ts is the slow quasi-transverse partial wave modes. Thus, the magnitude of all six partial modes are represented in the same plot. It is observed that the magnitude of energy is maximum for the Tf mode in the middle layer while the other two modes are relatively weak. Also in the outer layer, there is an even energy distribution of the three mode types with Tf and Ll higher than the Ts partial mode. Using Fig. 3b, the orientation of these partial modes in the azimuthal (1-2 or x-y) plane is predicted. From this graph, the Tf in the middle layer is predicted to be at an angle of approximately 132 (or-42) degree which incidentally is along the fiber direction in the middle ply group and is also observed in the experimentally obtained plate wave flow pattern results (Fig. 3c). In the outer layers, the direction of Tf and Ll are both approximately equal to 40 degrees which again is almost along the fiber direction. This is again visible in the PWFP. The weak Ts is also visible, although not as clearly (due to low amplitude) along the 17 degree in the outer layers and -18 degree in the middle ply group as predicted by theory in Figure 3b.

For another specimen D $\{60_5/150_5\}_S$ shown in Fig. 4(a,b,c), again the theory predicts the plate wave flow pattern results. Here, the Tf partial mode at -25 degree azimuthal angle is the dominating partial mode in the middle ply group. Also, Ll partial mode is the predominant partial mode in the outer layers and propagates along +53 degree and has less energy when compared with the Tf partial mode in the middle layer. This is clearly manifested in the PWFP experiments and even the weak Ts partial mode along the -12 degree is visible. For specimen E $\{20_5/110_5\}_S$, the energy is primarily confined to Ll mode in the outer layers and is directed along the +19 degree azimuthal angle. The energy along the middle layers is extremely weak. This is again exhibited in the PWFP results.

(a)



(b)



(c)

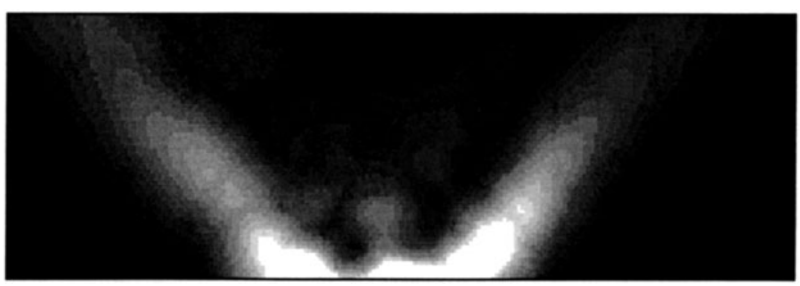


Figure 3. Results for specimen C $\{45_5/-45_5\}_S$, (a) Magnitude of Power Flow for the six bulk wave modes, (b) Orientation of the direction of energy flow in the 1-2 (x-y) plane, and (c) Plate Wave Flow Pattern at 90° to Transmitter.

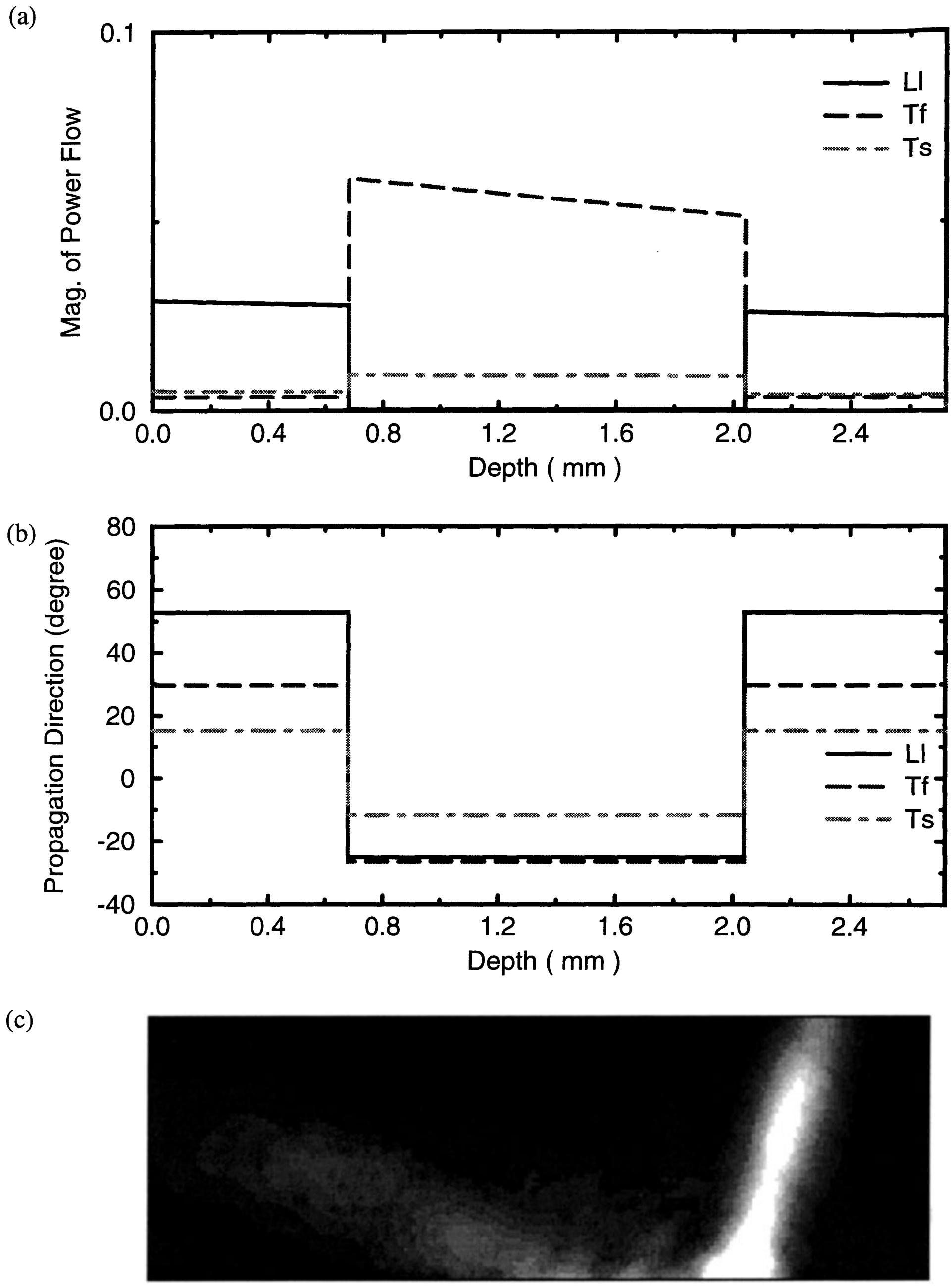


Figure 4. Results for specimen D$\{60_5/150_5\}_S$, (a) Magnitude of Power Flow for the six bulk wave modes, (b) Orientation of the direction of energy flow in the 1-2 (x-y) plane, and (c) Plate Wave Flow Pattern at 90° to Transmitter.

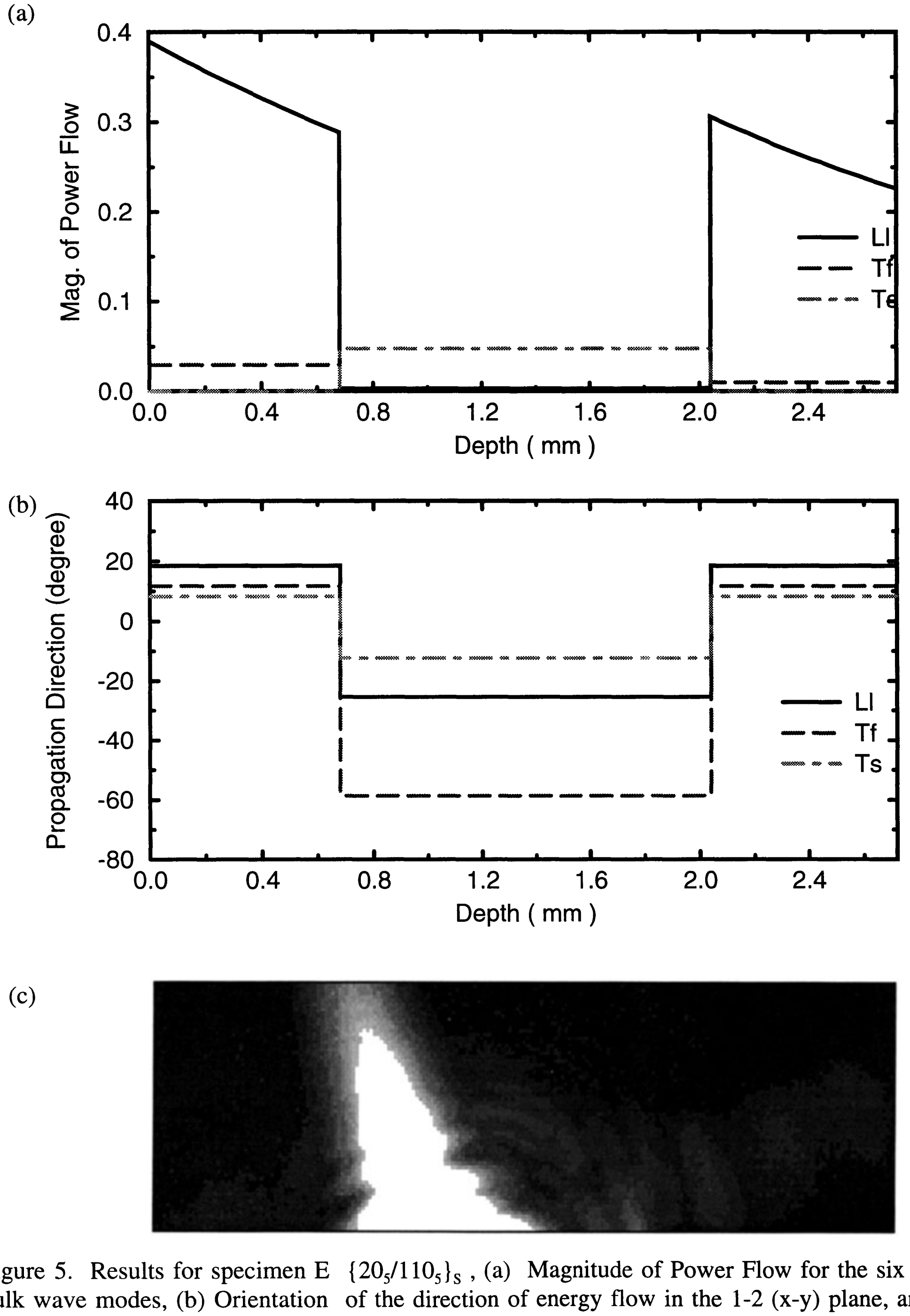


Figure 5. Results for specimen E $\{20_5/110_5\}_S$, (a) Magnitude of Power Flow for the six bulk wave modes, (b) Orientation of the direction of energy flow in the 1-2 (x-y) plane, and (c) Plate Wave Flow Pattern at 90° to Transmitter.

From these results, it is shown that the power flow vector for individual partial waves (without using superposition principle) can be used to explain guided wave behavior in inhomogeneous anisotropic multi-layered plates such as graphite-epoxy composites. This could be explained by the fact that the ultrasonic wave modes in the "zone of plate wave inception", when it encounters off axis (plane of incidence not along material symmetrical axes), tends to separate into partial modes along different directions (especially azimuthal to the material axis) and hence cannot interfere (or superpose) with each other. It can also be hypothetically inferred using this study that these partial wave modes are then responsible for the launching of guided plate waves in the composite specimen. It must be noted that the Plate Wave Flow Patterns were produced using a frequency*thickness product around 2.75 mm.MHz. and hence represent guided plate wave regime. [9-11]

ACKNOWLEDGEMENT

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